

11. cvičení - výsledky

Příklad 1.

- (a) $\frac{1}{2} \log |\cos x - 1| - \frac{1}{2} \log |\cos x + 1|$ pro $x \in (k\pi, (k+1)\pi), k \in \mathbb{Z}$.
- (b) $\frac{1}{\sqrt{2}} \arctan(\sqrt{2} \tan x + 1) + \frac{1}{\sqrt{2}} \arctan(\sqrt{2} \tan x - 1)$.
- (c) $\frac{1}{2} \sin^2 x + 2 \sin x + 3 \log(2 - \sin x)$.
- (d) $\frac{2}{\sqrt{3}} \arctan(\sqrt{3} \tan \frac{x}{2})$.
- (e) $\frac{1}{\sqrt{2}} \arctan(\sqrt{2} \tan x)$.
- (f) $\log |\tan x| - \frac{1}{2 \sin^2 x}$ pro $x \in (k\frac{\pi}{2}, (k+1)\frac{\pi}{2}), k \in \mathbb{Z}$.
- (g) $x + \frac{2}{1 + \tan \frac{x}{2}}$ pro $x \in (-\frac{\pi}{2} + k2\pi, \frac{3\pi}{2} + k2\pi), k \in \mathbb{Z}$.
- (h) $\log(3 + \tan^2 \frac{x}{2}) - \log(1 + \tan^2 \frac{x}{2}) + \frac{4}{\sqrt{3}} \arctan(\frac{1}{\sqrt{3}} \tan \frac{x}{2})$.
- (i) $\cos x - \frac{5}{2} \arctan(\frac{1}{2} \cos x)$.
- (j) $\frac{4}{\sqrt{3}} \arctan(\frac{1}{\sqrt{3}} \tan x) - x$ pro $x \in (-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi), k \in \mathbb{Z}$.

Příklad 2.

- (a) $\frac{1}{2} \log |\tan x - 1| - \frac{1}{4} \log(\tan^2 x + 1) + \frac{1}{2} x$ pro $(-\frac{\pi}{2} + k\pi, \frac{\pi}{4} + k\pi), (\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi), k \in \mathbb{Z}$.
- (b) $\frac{1}{6} \log |\sin^2 - \sin x + 1| - \frac{1}{3} \log(1 + \sin x) + \frac{1}{\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}}(2 \sin x - 1)\right)$.
- (c) $x - \frac{1}{\sqrt{2}} \arctan(\sqrt{2} \tan x)$.
- (d) $\frac{1}{\sqrt{5}} \arctan\left(\frac{1}{\sqrt{5}}(3 \tan \frac{x}{2} + 1)\right)$.
- (e) $\log |2 + \tan x|$.

Příklad 3.

- (a) $-\log x - \log |\log x - 1|$ pro $x \in (0, e) \cup (e, \infty)$.
- (b) $-\frac{1}{2} x + \frac{1}{3} \log |e^x - 1| + \frac{1}{6} \log(e^x + 2)$ pro $x \in \mathbb{R} \setminus \{0\}$.
- (c) $-2\sqrt{x} + 4\sqrt[4]{x} - 4 \log(1 + \sqrt[4]{x})$ pro $x > 0$.
- (d) $-x - 4\sqrt{2x+3} - 9 \log |\sqrt{2x+3} - 3| + \log(\sqrt{2x+3} + 1)$ pro $x \in (-\frac{3}{2}, -1) \cup (-1, 3) \cup (3, \infty)$.
- (e) $e^x + \frac{2}{3} \log |e^x - 1| - \frac{1}{3} \log(e^{2x} + e^x + 1)$ pro $x \in \mathbb{R} \setminus \{0\}$.
- (f) $\log |\log(\log x)|$.
- (g) $\log\left(1 + \sqrt{\frac{x+2}{x-3}}\right) - \log\left|\sqrt{\frac{x+2}{x-3}} - 1\right| - 2\sqrt{\frac{2}{3}} \arctan\left(\sqrt{\frac{3}{2}} \sqrt{\frac{x+2}{x-3}}\right)$.
- (h) $\frac{6}{7} \sqrt[6]{(x+1)^7} + \frac{6}{5} \sqrt[6]{(x+1)^5} - \frac{3}{2} \sqrt[6]{(x+1)^4} + 2 \sqrt[6]{(x+1)^3} - 3 \sqrt[6]{(x+1)^2} + 6 \sqrt[6]{x+1} - 6 \log(\sqrt[6]{x+1} + 1)$.

$$(i) \frac{1}{3} \log |\log x - 1| - \frac{1}{6} \log |\log^2 x + \log x + 1| - \frac{1}{\sqrt{3}} \arctan \left(\frac{2}{\sqrt{3}} (\log x + \frac{1}{2}) \right).$$

$$(j) -\frac{3}{2} \log \left(1 + \sqrt{\frac{x-1}{x+2}} \right) + \frac{3}{2} \log \left| 1 - \sqrt{\frac{x-1}{x+2}} \right| + \frac{3}{2} \cdot \frac{1}{1 - \sqrt{\frac{x-1}{x+2}}} - \frac{3}{2} \cdot \frac{1}{1 + \sqrt{\frac{x-1}{x+2}}}.$$

$$(k) \log \left| \sqrt{1 - \frac{2}{x}} - 1 \right| - \log \left(\sqrt{1 - \frac{2}{x}} + 1 \right) - \frac{1}{1 + \sqrt{1 - \frac{2}{x}}} + \frac{1}{1 - \sqrt{1 - \frac{2}{x}}}.$$

$$(l) 6 \log \sqrt[6]{x} - \frac{3}{2} \log(\sqrt[6]{x} + 1) - \frac{9}{4} \log(2\sqrt[3]{x} - \sqrt[6]{x} + 1) - \frac{3}{2\sqrt{7}} \arctan \left(\frac{4\sqrt[6]{x}-1}{\sqrt{7}} \right).$$

Příklad 4. Vzorové řešení: http://www.karlin.mff.cuni.cz/~kuncova/1920LS_MA2/16a_reseni.pdf

$$(a) \frac{-2-4\sqrt[4]{x}}{(1+\sqrt[4]{x})^2} \text{ (př. 1(c))}$$

$$(b) 2 \arctan \sqrt{\frac{1-x}{1+x}} - \log \left| \frac{1+\sqrt{\frac{1-x}{1+x}}}{1-\sqrt{\frac{1-x}{1+x}}} \right| \text{ (pr. 2(c))}$$

$$(c) -\frac{2}{\sqrt{5}} \log \left| 2\sqrt{\frac{2-x}{x+1}} + 3 + \sqrt{5} \right| + \frac{2}{\sqrt{5}} \log \left| -2\sqrt{\frac{2-x}{x+1}} - 3 + \sqrt{5} \right| - 2 \arctan \sqrt{\frac{2-x}{x+1}} \text{ (př. 3 (b))}$$

$$(d) \log \frac{\left(1+\sqrt{\frac{x+1}{x-1}}\right)^2}{1+\left(\sqrt{\frac{x+1}{x-1}}\right)^2} - \frac{2\sqrt{\frac{x+1}{x-1}}}{1+\left(\sqrt{\frac{x+1}{x-1}}\right)^2} \text{ (př. 2(a))}$$

$$(e) -\frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x^2+x+1}-x+1+\sqrt{3}}{x-\sqrt{x^2+x+1}-1+\sqrt{3}} \right| \text{ (př. 3(c))}$$

$$(f) \frac{1}{2} \left(\sqrt{x^2+2x+4} - x + 2 \log \left| \sqrt{x^2+2x+4} - x - 1 \right| + \frac{3}{\sqrt{x^2+2x+4}-x-1} \right) \text{ (př. 3 (a))}$$